Engineering Mathematics II (EM-II)

Endsem Preparation: Most Repetitive & Probable Questions

(Syllabus Adapted: Includes Harmonic Analysis, Excludes Z-Transform)

Q1. Fourier Series & Harmonic Analysis

Based on the historical data, this section heavily features standard polynomial expansions and harmonic analysis from data tables.

Type 1: Full Range Fourier Series

Expand $f(x) = x^2 - x$ or $f(x) = \pi x - x^2$ in the interval $[0, 2\pi]$ or $[-\pi, \pi]$.

Type 2: Half-Range Series

Obtain the half-range sine or cosine series for $f(x) = \pi x - x^2$ or $f(x) = x^2$ in the interval $0 \le x \le \pi$.

Type 3: Harmonic Analysis (Very High Probability)

Using Fourier series, expand y up to the first or second harmonic for the given data table (usually 6 pairs of (x, y) covering 0 to $360°$ or 0 to 2π).

Key Formulas Required:

- **Euler's Formulae:** $a_0 = 1/\pi \int f(x)\,dx$, $a_n = 1/\pi \int f(x)\cos(nx)\,dx$, $b_n = 1/\pi \int f(x)\sin(nx)\,dx$
- **Harmonic Analysis:** $a_0 = 2/N \Sigma y$
- $a_n = 2/N \Sigma y \cdot \cos(n\theta)$
- $b_n = 2/N \Sigma y \cdot \sin(n\theta)$
- **Fourier Series:** $f(x) = a_0/2 + \Sigma (a_n \cos(nx) + b_n \sin(nx))$

Q2. Reduction, Gamma, Beta Functions & DUIS

This is the most strictly patterned section. Expect exactly one question from each of the following four subtopics.

Type 1: DUIS (Differentiation Under Integral Sign)

Using DUIS rule, prove that $\int_0^\infty (e^{-ax} - e^{-bx})/x\,dx = \log(b/a)$ or $\int_0^1 (x^a - x^b)/\log(x)\,dx = \log((a+1)/(b+1))$.

Type 2: Gamma Function

Evaluate $\int_0^\infty x^9 e^{-2x^2}\,dx$ or $\int_0^\infty x^4/4^x\,dx$ using the Gamma function definition.

Type 3: Beta Function

Evaluate $\int_3^7 (x-3)^{1/4}(5-x)^{1/4}\,dx$ or $\int_0^1 x^5(1-\sqrt{x})^{5/2}\,dx$.

Type 4: Reduction Formula

If $I_n = \int_0^{\pi/4} \cos^{2n}(x)\,dx$, prove the reduction formula $I_n = 1/(n\,2^{n+1}) + (2n-1)/(2n)\,I_{n-1}$.

Key Formulas Required:

- **Leibnitz's Rule (DUIS):** $d/da\,[\int f(x,a)\,dx] = \int \partial/\partial a\, f(x,a)\,dx$
- **Gamma:** $\Gamma(n) = \int_0^\infty e^{-x} x^{n-1}\,dx$
- **Beta:** $\beta(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$
- **Relation:** $\beta(m, n) = \Gamma(m)\Gamma(n)/\Gamma(m+n)$
- **Transformation:** $\int_0^{\pi/2} \sin^p(\theta)\cos^q(\theta)\,d\theta = \Gamma((p+1)/2)\,\Gamma((q+1)/2)/(2\Gamma((p+q+2)/2))$

Q3. Curve Tracing

This section requires sketching of Cartesian, Polar, and Parametric curves.

Type 1: Cartesian Curves

Sketch the Cissoid $y^2(2a - x) = x^3$ or $a y^2 = x^2(a - x)$.

Type 2: Polar Curves - Cardioid

Sketch the Cardioid $r = a(1 \pm \cos\theta)$ or $r = a(1 \pm \sin\theta)$.

Type 3: Polar Curves - Rose Curves

Sketch the multi-leaved rose $r = a\sin(2\theta)$ (4 leaves) or $r = a\cos(3\theta)$ (3 leaves).

Type 4: Parametric Curves

Sketch the Cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$.

Key Tracing Rules to Remember:

- **Symmetry (Cartesian):** Even powers of y means symmetric about x-axis.
- **Symmetry (Polar):** If replacing θ with $-\theta$ leaves the equation unchanged, it's symmetric about the initial line.
- **Tangents at Origin:** Equate lowest degree terms to zero.
- **Asymptotes:** Find limits as $x \to \infty$ or equate coefficient of highest power of x/y to zero.

Q4. Multiple Integrals & Applications

Focus on evaluating double/triple integrals and finding areas/volumes.

Type 1: Area of Polar Regions

Find the area of one loop of the curve $r = a\cos(2\theta)$ OR the total area between the cardioids $r = a(1 + \cos\theta)$ and $r = a(1 - \cos\theta)$.

Type 2: Volume of Solid Regions

Find the volume of the tetrahedron bounded by the coordinate planes and the plane $x/a + y/b + z/c = 1$.

Type 3: Evaluating Double Integrals

Evaluate $\iint y\,dx\,dy$ over the region bounded by $y = x^2$ and $x + y = 2$.

Key Formulas Required:

- **Area (Cartesian):** $A = \iint dx\,dy$
- **Area (Polar):** $A = 1/2 \int r^2\,d\theta$
- **Volume:** $V = \iiint dx\,dy\,dz$
- **Change to Polar Coordinates:** Put $x = r\cos\theta$, $y = r\sin\theta$, and $dx\,dy = r\,dr\,d\theta$.

Analyzed and compiled based on past year Engineering Mathematics II examination papers.
Note: Z-Transform sections were strictly omitted as requested, prioritizing Harmonic Analysis under Q1.